Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Marking Scheme

Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Contents *Page*

Paper 1

Paper 2

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Mathematics

Ordinary Level

Paper 1

Solutions and Marking Scheme

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme
Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.
Marking scales – level descriptors
A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Summary of mark allocations and scales to be applied

Section A

Question 1	(25 marks)
(a)	10D
(b)	5C
(c)	10D
Question 2	(25 marks)
(a)(i)+(ii)	10D
(a)(iii)	5D
(b)	10C
Question 3	(25 marks)
(a)(i)	15D
(a)(ii)	5B
(b)	5C
Question 4	(25 marks)
(a)	10C
(b)	15D
Question 5	(25 marks)
(a)	15D
(b)	10C
Question 6	(25 marks)
(a)	10D
(b)	15C

Section B

Question 7	(55 marks)
(a)	5B
(b)(i)+(ii)	20D
(c)(i)	5C
(c)(ii)	5C
(d)(i)	5B
(d)(ii)	10C
(d)(iii)	5C
Question 8	(55 marks)
(a)(i)	5C
(a)(ii)	10C
(b)(i)	5C
(b)(ii)	10C
(b)(iii)	5C
(b)(iv)	5C
(b)(v)	10C
(b)(vi)	5C
Question 9	(40 marks)
(a)	5B
(b)	5C
(c)(i)	5B
(c)(ii)	10C
(c)(iii)	10D
(d)	5C

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Throughout the scheme indicate by use of # where an arithmetic error occurs.

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✖	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
#	Hash	Rounding error Unit error Arithmetic error Misreading	
〰	Horizontal wavy	Error	
✔ₗ	Tick L		The work presented in the body of the script merits low partial credit
✔ₘ	Tick M		The work presented in the body of the script merits mid partial credit
✔ₕ	Tick H		The work presented in the body of the script merits high partial credit
[	Left Bracket		Another version of this solution is presented elsewhere and is worth equal or higher credit
⑂	Vertical wavy	No work on this page (portion of the page)	
▭o▭	Oversimplify	The candidate has oversimplified the work	

Note: It may be necessary to use a combination of 2 symbols in the right margin to clearly show your judgement of the work in the body of the script:

✔ #   must be used to signify that Full Credit – 1 is merited by the work presented

[✔ₘ   Signifies that the work in the body of the script is worth mid partial credit but another effort at the work has been awarded this or higher credit

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where $\#$ and $\boxed{\sim\sim}$ and $\boxed{\sim\sim}$ appear in the body of the work then ✔I should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then ✔m should be placed in the right margin.

A ✔ in the body of the work may sometimes be used indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

| Q1	Model Solution – 25 Marks	Marking Notes
(a)	$35300 \times 0 \cdot 2 = €7060$ $4700 \times 0 \cdot 4 = €1880$ $€7060 + 1880 = €8940$ $€8940 - 1650 = €7290$ net tax	**Scale10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Use of 20%, 40% or similar • Finds 4700 *Mid Partial Credit*: • Both taxes formulated • One tax found correctly *High Partial Credit:* • Both taxes found and added
(b)	$40000 - (7290 + 1500) = €31210$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Brings down answer from (a) • Finds 8790 *High Partial Credit:* • Income formulated correctly • Correct answer without work
(c)	$35300 \times 0 \cdot 2 + 2700 \times 0 \cdot 4 = 8140$ $8140 - 1650 = 6490$ $38000 - 6490 = 31510$ $31510 - 31210 = €300$ $\Rightarrow €300$ increase	**Scale10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Use of 7060 • Finds 2700 *Mid Partial Credit:* • Finds 8140 or equivalent *High Partial Credit:* • Finds 6490 and 31510 or equivalent

Q2	Model Solution – 25 Marks	Marking Notes
(a) **(i)** **+** **(ii)**	$z_2 = 2(2 + i) = 4 + 2i$ $\bar{z}_1 = 2 - i$ 	**Scale10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Correct substitution into $2z_1$ • Finds z_2 or $\bar{z}_1$ correctly and stops *Mid Partial Credit:* • Finds $4 + 2i$ and $2 - i$ • z_2 or $\bar{z}_1$ found correctly and plots, with or without label *High Partial Credit:* • z_2 and $\bar{z}_1$ found and at least one plotted correctly, without labels • Mixes up axes but finds, plots and labels correctly **Note:** Ignore extra incorrect plotting
(a) **(iii)**	$$|z_2| = \sqrt{16 + 4} = \sqrt{20}$$ $$|z_1 + \bar{z}_1| = 4 \neq \sqrt{20}$$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Correct formula • Some correct substitution • Finds $z_1 + \bar{z}_1$ *Mid Partial Credit:* • Finds $|z_2|$ or $|z_1 + \bar{z}_1|$ *High Partial Credit:* • Finds $|z_2|$ and $|z_1 + \bar{z}_1|$ but no conclusion or incorrect conclusion

Q2	Model Solution – 25 Marks	Marking Notes
(b)	$$(2 + i)^2 - 4(2 + i) + 5 = 0$$ $$4 + 4i + i^2 - 8 - 4i + 5 = 0$$ $$9 - 9 + 4i - 4i = 0$$ or $$\frac{4 \pm \sqrt{16 - 4(1)5}}{2}$$ $$= \frac{4 \pm \sqrt{-4}}{2}$$ $$= 2 \pm i$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some relevant substitution *High Partial Credit:* • Full substitution with correct multiplication of $(2 + i)^2$ or $-4(2 + i)$ **Note**: failure to draw a relevant conclusion from incorrect work to be treated as an error or **Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some relevant substitution into quadratic roots formula • Quadratic roots formula written *High Partial Credit:* • Full substitution with some errors but with complex roots and relevant conclusion given • Gets $\frac{4 \pm \sqrt{-4}}{2}$ **Note**: failure to draw a relevant conclusion from incorrect work to be treated as an error

3	**Model Solution – 25 Marks**			**Marking Notes**		

(a)

(i)

x	−1	0	1	2	3	4
$f(x)$	2	-2	2	8	10	2
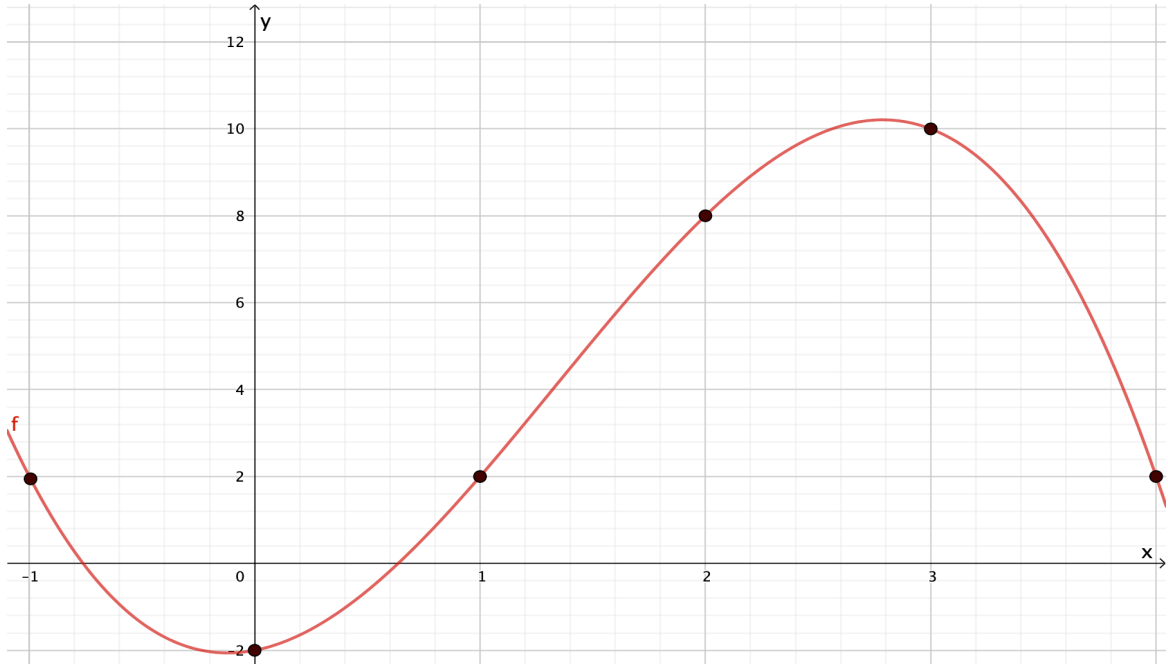
Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:
- Relevant work towards finding a value for $f(x)$
- One point found correctly
- One point from candidates table plotted correctly

Mid Partial Credit:
- Table fully correct
- All points from candidates incorrect table plotted and/or joined correctly

High Partial Credit:
- All correct points found and plotted correctly, but not joined or joined incorrectly
- 4 or 5 correct points on table plotted correctly with joining

Q3	Model Solution – 25 Marks	Marking Notes
(a)(ii)	Roots $\approx -0{\cdot}7, 0{\cdot}7$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* • One correct/relevant root given or indicated on graph
(b)	$f'(x) = -3x^2 + 8x + 1$ $f''(x) = -6x + 8 = 0$ $x = \dfrac{4}{3}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Any correct differentiation • $f''(x)$ found from non-quadratic • $f'(x)$ *High Partial Credit:* • $f''(x)$ found

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$$6x + 2 + 5x - 10 = 47$$ $$x = 5$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • 10 identified as common denominator • Some relevant multiplication • $2(3x + 1) + 5(x - 2)$ without CD • Tests a value/s for x not equal to 5 *High Partial Credit:* • $6x + 2 + 5x - 10 = 47$ or equivalent **Note:** Accept Trial and Improvement method for $x = 5$ verified for full marks
(b)	$$x = 5y - 13$$ $$(5y - 13)^2 + y^2 = 13$$ $$25y^2 - 130y + 169 + y^2 - 13 = 0$$ $$26y^2 - 130y + 156 = 0$$ $$y^2 - 5y + 6 = 0$$ $$(y - 2)(y - 3) = 0$$ $$y = 2 \text{ and } y = 3$$ $$\implies x = -3 \text{ and } x = 2$$ $$(-3, 2) \text{ and } (2, 3)$$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* • Isolates one variable or transposes correctly *Mid Partial Credit:* • $(5y - 13)^2 + y^2 = 13$ or equivalent equation in one variable *High Partial Credit:* • One relevant value (root) found from quadratic

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$A = \dfrac{3}{2}[0+0+2(15+18+15+12+15+18+15]$ $A = 324 \text{ m}^2$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* • Any correct dimension identified • Correct answer without work *Mid Partial Credit* • Correct formula with some correct substitution *High Partial Credit:* • Correct formula fully substituted correctly
(b)	$1 \cdot 6 \times 60 \times 60 \div 1000 = 5 \cdot 76$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $1 \cdot 6 \times 60$ or equivalent • Any work of merit *High Partial Credit:* • $1 \cdot 6 \times 60 \times 60 = 5760$ • $1 \cdot 6 \times 60 \times 60 \div 1000$ not finished

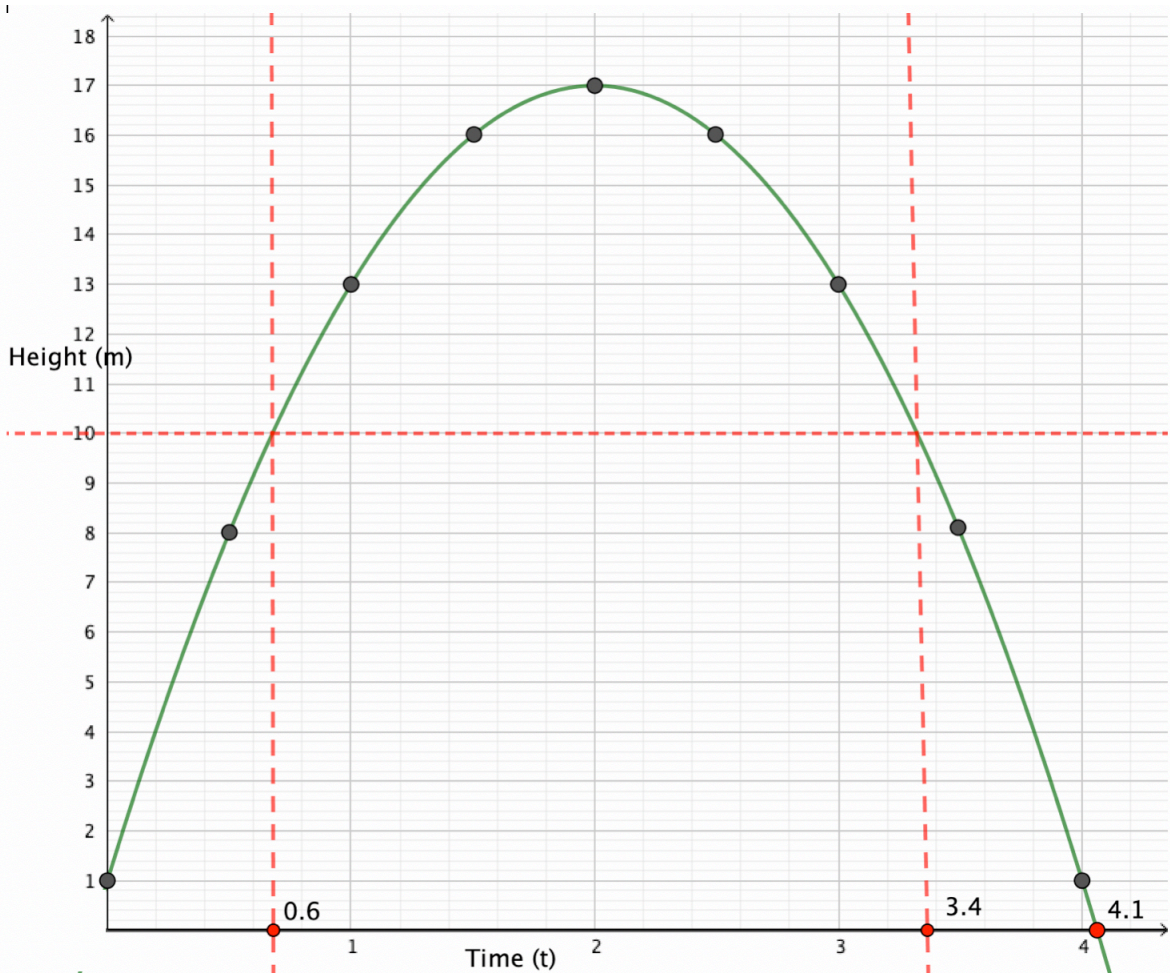| Q6	Model Solution – 25 Marks	Marking Notes
(a)	$$6 - 2x < 8$$ $$2x > -2$$ $$x > -1$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Any correct multiplication *Mid Partial Credit:* • Correct transposing from correct work *High Partial Credit:* • $x > -1$ but no plot or incorrectly plotted • $x < -1$ and correctly plotted
(b)	$$2^{2x-1} = 2^6$$ $$\Rightarrow 2x - 1 = 6$$ $$2x = 7$$ $$x = \frac{7}{2}$$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • 2^6 • List of powers of 2 • Correct answer no work *High Partial Credit:* • Equation in x, $2x - 1 = 6$ *Zero Credit:* • $4x - 2 = 64$

Section B

Q7	Model Solution – 55 Marks	Marking Notes
(a)	$$f(0) = 1$$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* • $f(0)$ • $t = 0$
(b) **(i)** **&** **(ii)**	**Table and Graph (20 marks)**	

Time (*t*)	0	0·5	1	1·5	2	2·5	3	3·5	4
Height (m)	1	8	13	16	17	16	13	8	1

Q7	Model Solution – 55 Marks	Marking Notes
(b) **(i)** **&** **(ii)**		**Scale 20D (0,5,10,15,20)** *Low Partial Credit*: • Up to 6 correct entries *Mid Partial Credit*: • Between 7 and 12 correct entries *High Partial Credit:* • 13 to 17 correct entries, no joining or incorrect joined with a straight line **Note**: An entry is a calculation or a plot
(c) **(i)**	$4 \cdot 1$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $4 \cdot 1$ or similar, but no extension shown on graph • Extension to graph only *High Partial Credit:* • Clear indication on graph, but no value given
(c) **(ii)**	Work to be shown on graph. $3 \cdot 4 - 0 \cdot 6 = 2 \cdot 8$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $f(3 \cdot 4)$ found or indicated on graph • $f(0 \cdot 6)$ found or indicated on graph *High Partial Credit:* • $3 \cdot 4 - 0 \cdot 6$ or equivalent and fails to finish

Q7	Model Solution – 55 Marks	Marking Notes
(d) **(i)**	$$f'(t) = -8t + 16$$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* • Any correct differentiation
(d) **(ii)**	$$-8(4{\cdot}0) + 16 = -16$$ Speed $=16$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some relevant substitution into candidate's $f'(x)$ • Brings down $f'(x)$ to this part *High Partial Credit:* • Candidate's $f'(x)$ fully substituted, but not worked out **Note**: Accept -16
(d) **(iii)**	$$-8t + 16 = -8$$ $$8t = 24$$ $$t = 3$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Candidate's $f'(x) = z$ where $z \neq 8$ or -8 *High Partial Credit:* • Candidate's $f'(x) = 8$ finished correctly where the solution is viable • $-8t + 16 = -8$ and fails to finish

Q8	Model Solution – 55 Marks	Marking Notes
(a) (i)	$$P = \frac{480 \times 2500}{5252}$$ $$= 228 \cdot 48$$ $$= 228$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some relevant substitution into formula *High Partial Credit:* • Formula fully substituted
(a) (ii)	$$R = \frac{5252\,P}{T}$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Any relevant transposing *High Partial Credit:* • Transposing complete but with one error

(b) (i)

Month	1	2	3	4	5	6
Profit (€)	−4000	−3750	**−3500**	−3250	**−3000**	**−2750**

Scale 5C (0, 2, 3, 5)
• One additional correct entry

High Partial Credit:
• Two correct entries

(b) (ii)	$$T_n = -4000 + (n-1)250$$ $$T_n = 250n - 4250$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Identifies correct a and/or d • Writes $T_n = a + (n-1)d$ *High Partial Credit:* • Formula fully substituted

Q8	Model Solution – 55 Marks	Marking Notes
(b) **(iii)**	$$T_{25} = 250(25) - 4250$$ $$= €2000$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some relevant substitution into formula • Month by month method incomplete *High Partial Credit:* • Formula fully substituted but fails to finish
(b) **(iv)**	$$T_n = 0$$ $$250n - 4250 = 0$$ $$n = 17$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some relevant substitution into formula • Month by month method incomplete • $n = 17$ without work • $T_n = 0$ written *High Partial Credit:* • $250n - 4250 = 0$
(b) **(v)**	$$S_n = \frac{n}{2}[2(-4000) + (n-1)250]$$ $$S_n = \frac{n}{2}[-8000 + 250n - 250]$$ $$S_n = \frac{n}{2}[-8250 + 250n]$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Identifies a and/or d • Correct formula for S_n *High Partial Credit:* • Formula fully substituted correctly
(b) **(vi)**	$$S_{37} = \frac{37}{2}[-8250 + 250(37)]$$ $$= 18500$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some relevant substitution into either S_n formula *High Partial Credit:* • Correct formula fully substituted but work not complete

Q9	Model Solution – 40 Marks	Marking Notes
(a)	$3000(1\cdot8)^8 = 330598\cdot817$ $\Rightarrow 330598$ or 330599	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* • Some relevant substitution into formula
(b)	$31493 \times 1\cdot8 = 56687\cdot4$ $= 56687$ or $3000(1\cdot8)^4 = 31493$ $3000(1\cdot8)^5 = 56687$	**Scale 5C (0, 2, 3, 5)** Low Partial Credit: • Some relevant substitution into formula • Incomplete trial and improvement to find m *High Partial Credit:* • $m = 4$ found by trial and improvement but multiplied by $1\cdot8$ incorrectly • $3000(1\cdot8)^5$
(c) (i)	$600000 \times 0\cdot0012 = €720$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* Use of 600000 and/or 0·0012 • Decimal error
(c) (ii)	$\dfrac{1285\cdot37}{0\cdot0012} = 1071141\cdot667$ $= 1071142$ or 1071141	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit*: • $1285\cdot3 \times 0\cdot0012$ *High Partial Credit:* • $1285\cdot37 \div 0\cdot0012$ • $1285\cdot37 \times 0\cdot0012$ or equivalent $= 1\cdot5$

Q9	Model Solution – 40 Marks	Marking Notes
(c) (iii)	**Month 4:** $$3000(1{\cdot}8)^4 = 31492{\cdot}8$$ $$31492{\cdot}8 \times 0{\cdot}0012 = €37{\cdot}79$$ $$37{\cdot}79 - 80 = €42{\cdot}21 \text{ Loss}$$ **Month 12:** $$3000(1{\cdot}8)^{12} = 3470494{\cdot}144$$ $$3470494{\cdot}144 \times 0{\cdot}0012 = €4164{\cdot}59$$ $$4164{\cdot}59 - 80 = €4084{\cdot}59 \text{ Gain}$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Either month formulated correctly • Finds number of users for either month *Mid Partial Credit* • Income for either month evaluated correctly • Finds number of users for both months *High Partial Credit:* • Incomes for both months evaluated correctly • Profit/Loss for one month evaluated correctly
(d)	$$\text{Exchange Rate} = \frac{55}{62{\cdot}70}$$ $$\text{Exchange Rate} = 0{\cdot}8772$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $\frac{62{\cdot}7}{55}$ *High Partial Credit:* • $\frac{62{\cdot}7}{55} = 1{\cdot}14$ • $\frac{55}{62{\cdot}7}$ or $\frac{50}{57}$ and stops

Blank Page

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2019

Mathematics

Ordinary Level

Paper 2

Solutions and Marking Scheme

300 marks

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme
Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 4, 6, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 9, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors
A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Summary of mark allocations and scales to be applied

Section A			*Section B*	
Question 1	(25 marks)		Question 7	(50 marks)
(a)	10C		(a)	10C
(b)	5C		(b)	10C
(c)(i)	5C		(c)(i)	15C
(c)(ii)	5C		(c)(ii)	5A
			(c)(iii)	5C
Question 2	(25 marks)		(d)	5C
(a)	10C			
(b)	5C		Question 8	(45 marks)
(c)	5B		(a)(i)	10C
(d)	5D		(a)(ii)	5C
			(a)(iii)	10C
Question 3	(25 marks)		(b)(i)	10C
(a)	10B		(b)(ii)	5C
(b)	5C		(b)(iii)	5C
(c)	5C			
(d)	5C		Question 9	(55 marks)
			(a)	15C
Question 4	(25 marks)		(b)	10C
(a)	5C		(c)	10C
(b)	5C		(d)(i)	5D
(c)	5C		(d)(ii)	10C
(d)	10C		(d)(iii)	5D
Question 5	(25 marks)			
(a)	10D			
(b)	15D			
Question 6	(25 marks)			
(a)(i)	15D			
(a)(ii)	5C			
(b)	5D			

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale* 10C, 9 marks may be awarded.
Rounding and units penalty to be applied only once in each part (a), (b), (c) etc.
Throughout the scheme indicate by use of **#** where an arithmetic error occurs.

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Section A

Q1	Model Solution – 25 Marks	Marking Notes
(a)		**Scale 10C (0, 4, 6, 10)**
		1 8 8 9							*Low Partial Credit:*
		2 0 1 1 2							• Attempt at ordering ages
		3 2 3 3 4 4 5 9		• One correct entry
		4 1 2 3 3 9					
		5 4 5 7 7 8					*High Partial Credit:*
		6 3 3 4 5						• 15 correct entries
		KEY: 1 | 9 = 19 years of age	
		Full Credit:
		• Correct answer without work
(b)	$$\frac{14}{28}$$ $$= 50\%$$	**Scale 5C (0, 2, 3, 5)**
		Low Partial Credit:
		• Work of merit
		• Mention of 14 or 28
		• Reference to × by 100
		High Partial Credit:
		• Answer given as $\frac{14}{28}$ or equivalent
		Full Credit:
		• Correct answer without work

Q1	Model Solution – 25 Marks	Marking Notes
(c)(i)	$$\frac{3}{28}$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • $\frac{x}{28}, (x \neq 3, x \leq 28)$ • $\frac{3}{x}, (x \neq 28, x \geq 3)$ *Full Credit:* • Correct answer without work *Zero Credit:* • Probability > 1, without work
(c) (ii)	$$= \frac{3}{28} + \frac{4}{28}$$ $$= \frac{7}{28}$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • Both probabilities calculated correctly *Full Credit:* • Correct answer without work *Zero Credit:* • Probability > 1, without work

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ $$= \frac{5 - 2}{8 - 4}$$ $$= \frac{3}{4}$$	**Scale 10C(0, 4, 6, 10)** *Low Partial Credit:* • Work of merit • Formula with some correct substitution *High Partial Credit:* • Formula substituted correctly *Full Credit:* • Correct answer without work *Misreading:* • Finds slope of *PR* or *QR*
(b)	$$y - y_1 = m(x - x_1)$$ $$y - 2 = \frac{3}{4}(x - 4)$$ $$3x - 4y - 4 = 0$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Formula with some correct or consistent substitution *High Partial Credit:* • Formula substituted correctly or consistently • One incorrect substitution followed by correct solution • Answer not in the required format **Note:** Accept $$-3x + 4y + 4 = 0$$
(c)	$$-\frac{4}{3}$$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* • Work of merit *Full Credit:* • Correct answer without work

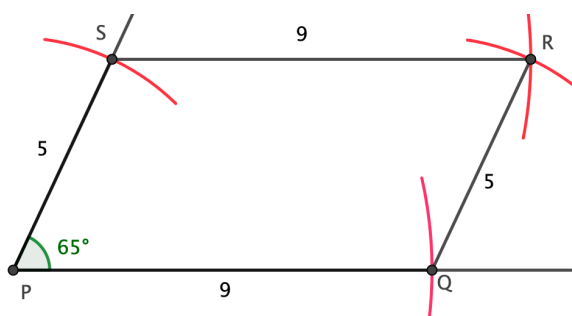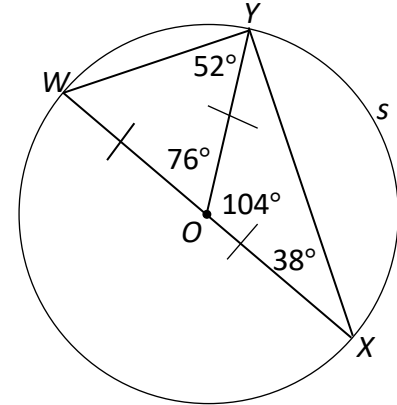| Q2	Model Solution – 25 Marks	Marking Notes
(d)	$$(4, 2) \ (8, 5) \ (2, 11)$$ $$\downarrow \qquad \downarrow \qquad \downarrow$$ $$(0, 0) \ (4, 3) \ (-2, 9)$$ $$= \frac{1}{2}|x_1 y_2 - x_2 y_1|$$ $$= \frac{1}{2}|(4)(9) - (-2)(3)|$$ $$= \frac{1}{2}|36 + 6|$$ $$= 21 \text{ square units}$$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Translation of any relevant point to (0,0) • Area of triangle formula with some substitution *Mid Partial Credit:* • Area of triangle formula substituted fully with 2 from P,Q, and R *High Partial Credit:* • Formula substituted correctly with translated points • Area of triangle found from formula substituted fully with 2 from *P,Q, R* • An incorrect translation applied correctly

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$$1 - 0{\cdot}7 = 0{\cdot}3$$	**Scale 10B (0, 5, 10)** *Mid Partial Credit:* • Work of merit *Full Credit:* • Correct answer without work *Zero Credit:* • Probability > 1, without work
(b)	$$0{\cdot}7 \times 0{\cdot}7 = 0{\cdot}49$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • Correct answer indicated but not calculated • $0{\cdot}3 \times 0{\cdot}3 = 0{\cdot}09$ *Full Credit:* • Correct answer without work *Zero Credit:* • Probability > 1, without work
(c)	$$0{\cdot}7 \times 0{\cdot}7 \times 0{\cdot}3 \times 3$$ $$= 0{\cdot}147 \times 3$$ $$= 0{\cdot}441$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, e.g. List given *High Partial Credit:* • Correct answer indicated but not calculated *Full Credit:* • Correct answer without work *Zero Credit:* • Probability > 1, without work

Q3	Model Solution – 25 Marks	Marking Notes
(d)	$$(0\cdot3)^3 \times 0\cdot7$$ $$= 0\cdot0189$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, e.g. Correct list *High Partial Credit:* • Correct answer indicated but not calculated *Full Credit:* • Correct answer without work *Zero Credit:* • Probability > 1, without work

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$$S(-4, 11) \rightarrow P(2, 3)$$ $$\Rightarrow x \uparrow 6 \text{ and } y \downarrow 8$$ $$R(4, 17) \rightarrow Q(10, 9)$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit • Translation identified or indicated on the diagram *High Partial Credit:* • One co-ordinate correct • An incorrect translation applied correctly • Correct translation applied to an incorrect point *Full Credit:* • Correct answer without work
(b)	Mid-point of *PR* $$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$ $$= \left(\frac{2 + 4}{2}, \frac{3 + 17}{2} \right)$$ $$= (3, 10)$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Formula with some relevant substitution • Indicates the centre on the diagram *High Partial Credit:* • Formula substituted correctly *Full Credit:* • Correct answer without work
(c)	$$\text{Radius} = \frac{1}{2}|RS| \text{ or } \frac{1}{2}|SP|$$ $$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ $$= \sqrt{(4 + 4)^2 + (17 - 11)^2}$$ $$= \sqrt{64 + 36} = 10$$ $$r = \frac{1}{2}(10) = 5 \text{ units}$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Distance formula with some relevant substitution *High Partial Credit:* • Formula substituted correctly • Correct answer without work

Q4	Model Solution – 25 Marks	Marking Notes
(d)	Centre $= (3,10)$ Radius $= 5$ $$(x-3)^2 + (y-10)^2 = 25$$ or $$x^2 + y^2 - 6x - 20y + 84 = 0$$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* - Work of merit *High Partial Credit:* - Formula substituted with centre and radius i.e. $(x-3)^2 + (y-10)^2 = 5^2$ - One error in substitution and finished correctly *Full Credit:* - Correct or consistent answer without work **Note:** Accept work with candidates centre and radius

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$\text{Area}_L = \pi r^2$ $\qquad = \pi(3)^2$ $\qquad = 9\,\pi$ $\text{Area}_S = \pi r^2$ $\qquad = \pi(2{\cdot}5)^2$ $\qquad = 6{\cdot}25\,\pi$ $\text{Area}_C = 9\pi - 6{\cdot}25\pi$ $\qquad = 8{\cdot}64 \text{ cm}^2$ $\text{Perimeter}_L = 2\pi(3)$ $\qquad\quad = 6\pi$ $\text{Perimeter}_S = 2\pi(2{\cdot}5)$ $\qquad\quad = 5\pi$ $\text{Perimeter}_C = 6\pi + 5\pi$ $\qquad\quad = 34{\cdot}56 \text{ cm}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Area formula with some substitution or Circumference formula with some substitution *Mid Partial Credit:* • Area or perimeter of crescent worked correctly • All four formulae substituted correctly *High Partial Credit:* • Expression for Area and Perimeter substituted correctly with no conclusion • Incorrect operator (– or +) in either part but finishes correctly • Answers in terms of π
(b)	$\text{Volume} = \dfrac{1}{3}\pi(7)^2(12)$ $\qquad\quad = \dfrac{588}{3}\pi$ $\qquad\quad = 196\pi$ $\qquad\quad = 615{\cdot}7521601$ $\text{Time} = \dfrac{615{\cdot}7521601}{0{\cdot}5(1000)}$ $\qquad = 1{\cdot}23150432 \text{ minutes}$ $\qquad = 1{\cdot}23150432 \times 60$ $\qquad = 73{\cdot}89025$ $\qquad = 74 \text{ seconds}$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* • Work of merit • Volume formula with some substitution *Mid Partial Credit:* • Formula substituted correctly *High Partial Credit:* • Volume found correctly i.e. $\dfrac{588}{3}\pi$ or equivalent.

Q6	Model Solution – 25 Marks	Marking Notes
(a) (i)		**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* • A labelled pilot diagram drawn • One correct length or angle drawn *Mid Partial Credit:* • Any 2 components constructed correctly *High Partial Credit:* • Any three components constructed correctly
(a) (ii)	$$\text{Area} = ab\sin C$$ $$= 9(5)\sin 65^\circ$$ $$= 45(0{\cdot}906307787)$$ $$= 40{\cdot}78385042$$ $$= 40{\cdot}78 \text{ cm}^2$$ or $$\text{Area} = a \times h_\perp$$ $$= 9[(5)\sin 65^\circ]$$ $$= 9(4{\cdot}531538935)$$ $$= 40{\cdot}78385042$$ $$= 40{\cdot}78 \text{ cm}^2$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Formula with some substitution • Perpendicular distance between sides measured and used correctly to find area *High Partial Credit:* • Formula substituted correctly • Incorrect calculator mode (apply once only on paper)
(b)	 $\alpha = 52^\circ$ | $2\beta = 38^\circ$ $\beta = 19^\circ$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit • Pilot diagram *Mid Partial Credit:* • $\lvert\angle YOX\rvert = 104^\circ$ clearly stated or shown on the diagram • $\lvert\angle WYX\rvert = 90^\circ$ clearly stated or shown on the diagram *High Partial Credit:* • One value correct *Full Credit:* • Correct answers without work

37

Section B

Q7	Model Solution – 50 Marks	Marking Notes

(a)

Table B (€)				
	Meat	Fish	Milk, Cheese and Eggs	Fruits, Vegetables and Potatoes
Maximum	254	**177**	**175**	**172**
Minimum	54	**59**	**65**	**48**
Range	200	**118**	**110**	**124**

Scale 10C (0,4,6,10)
Low Partial Credit:
- Work of merit

High Partial Credit:
- Four correct entries

(b)

1st	2nd	3rd	4th	5th	6th	**7th**	8th	9th	10th	11th	12th	13th
48	62	77	86	95	104	**111**	116	125	136	137	150	172

Median = 111

Scale 10C (0, 4, 6, 10)
Low Partial Credit:
- Work of merit
- Effort at ordering data
- Median taken from an unordered list

High Partial Credit:
- All data ordered
- Correct median without work
- Correct median from an incorrect food category
- Correct median from a decreasing list

Q7	Model Solution – 50 Marks	Marking Notes
(c) (i)	$132 + 117 + 106 + 63 + 56 + 111 + 157$ $+54 + 85 + 131 + 254 + 78 + 112$ $\text{Sum} = 1456$ $\text{Mean} = \dfrac{1456}{13}$ $\text{Mean} = €112$	**Scale 15C (0, 5, 9, 15)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • Formula substituted correctly but not evaluated • Calculates the mean of an incorrect food category *Full Credit:* • Correct answer without work
(c) (ii)	$S.D. = €\,51$	**Scale 5A (0, 5)**
(c) (iii)	$112 + 51 = 163 \quad \& \quad 112 - 51 = 61$ $[61,163]$ Macedonia, Poland and Switzerland	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • Range found

Q7	Model Solution – 50 Marks				Marking Notes	

(d)

Table C (€)					
Country	Meat	Fish	Milk, Cheese and Eggs	Fruits, Vegetables and Potatoes	Total Cost
Ireland	36·04	31·32	24·32	31·28	122·96
Poland	18·36	**18·56**	**12·35**	**14·26**	**63·53**

Fish:

$$\frac{31\cdot32}{108} \times 64 = 18\cdot56$$

Milk, Cheese, Eggs:

$$\frac{24\cdot32}{128} \times 65 = 12\cdot35$$

Fruit, Veg, Potatoes:

$$\frac{31\cdot28}{136} \times 62 = 14\cdot26$$

Total Cost

$$= 18\cdot36 + \mathbf{18\cdot56} + \mathbf{12\cdot35} + \mathbf{14\cdot26}$$

$$= \mathbf{63\cdot53}$$

Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- Any work of merit

High Partial Credit:
- Two food categories filled correctly
- Correct answers without work

Q8	Model Solution – 45 Marks	Marking Notes
(a) (i)	$V = \dfrac{4}{3}\pi r^3$ $V = \dfrac{4}{3}\pi (3)^3$ $V = 36\pi \text{ cm}^3$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • Formula substituted correctly *Full Credit:* • Correct answer without work
(a) (ii)	$V = \pi r^2 h$ $\pi (5)^2 h = 36\pi$ $h = \dfrac{36\pi}{25\pi}$ $h = \dfrac{36}{25}$ cm or 1·44 cm	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit • Volume formula with some substitution *High Partial Credit:* • Formula substituted correctly and equation formed • One error in substitution in the equation and finishes correctly
(a) (iii)	C.S.A $= 2\pi rh$ $= 2\pi (5)(18)$ $= 180\pi$ Material left over $= 35 \times 20 - [180\pi]$ $= 700 - 565\cdot49$ $= 134\cdot51$ $= 134\cdot5 \text{ cm}^2$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit *High Partial Credit:* • 2 area formulae substituted correctly and stops • One error in substitution in getting the area and finishes correctly

Q8	Model Solution – 45 Marks	Marking Notes
(b) (i)	$$\frac{1}{\sqrt{800}} = 3 \cdot 5355$$ $$= 3 \cdot 54\%$$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Writes $\frac{1}{\sqrt{n}}$. • Sets up work as $\sqrt{800}$ *High Partial Credit:* • Formula substituted correctly • i.e.$\left(\frac{1}{\sqrt{800}}\right)$ • Answer as $0 \cdot 035355$ without work *Full Credit:* • Correct answer without work Special Case | **Without Work** | **Award** | | $3 \cdot 5355\%$ | *Full Credit −1* |
(b) (ii)	$$\hat{p} = \frac{350}{800} = 43 \cdot 75\%$$ 95% confidence interval: $$\left[\hat{p} - \frac{1}{\sqrt{n}} \, , \, \hat{p} + \frac{1}{\sqrt{n}}\right]$$ $$43 \cdot 75 - 3 \cdot 54 < p < 43 \cdot 75 + 3 \cdot 54$$ $$40 \cdot 21\% < p < 47 \cdot 29\%$$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit • $\hat{p} \pm \frac{1}{\sqrt{n}}$. *High Partial Credit:* • One boundary formed: • ($43 \cdot 75 - 3 \cdot 54$ or $43 \cdot 75 + 3 \cdot 54$) • Correct answer without work • Use of version of formula from HL • Answer in decimal form
(b) (iii)	$H_0 =$ the level of support is 50% $H_1 =$ the level of support is not 50% $$50\% \notin [40 \cdot 21\%, 47 \cdot 29\%]$$ 50% is not within the 95% CI so there is not enough evidence to support the claim. So, we reject H_0 and say that the level of support of the people in the area is not 50%	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Null hypothesis stated • C. I. written *High Partial Credit:* • Conclusion OK but reason incorrect or omitted • $50\% \notin [40 \cdot 21\%, 47 \cdot 29\%]$ with conclusion incorrect or omitted

Q9	Model Solution – 55 Marks	Marking Notes
(a)	$$\sin 30° = \frac{|QR|}{120}$$ $$|QR| = 120\sin 30°$$ $$|QR| = 60 \text{ km}$$	**Scale 15C (0, 5, 9, 15)** *Low Partial Credit:* • Work of merit • Correct trigonometric ratio or relevant formula with some substitution *High Partial Credit:* • Formula substituted correctly • One incorrect substitution and finished correctly • Incorrect calculator mode (apply once only on paper)
(b)	$$|RS|^2 = |RQ|^2 + |QS|^2$$ $$100^2 = 60^2 + |QS|^2$$ $$|QS|^2 = 10000 - 3600$$ $$|QS|^2 = 6400$$ $$|QS| = \sqrt{6400}$$ $$|QS| = 80 \text{ km}$$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit • Pythagoras formulated with some substitution • High Partial Credit: • Pythagoras substituted correctly • Answer as $\sqrt{6400}$ Correct answer without work
(c)	$$|PR|^2 = |QR|^2 + |PQ|^2$$ $$120^2 = 60^2 + |PQ|^2$$ $$|PQ|^2 = 10800$$ $$|PQ| = 103{\cdot}92304$$ $$|PS| = |PQ| - |QS|$$ $$|PS| = 103{\cdot}92 - 80$$ $$|PS| = 23{\cdot}92$$ $$|PS| \approx 24 \text{ km}$$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit • Pythagoras formulated with some substitution • $|PS| = |PQ| - 80$ *High Partial Credit:* • Pythagoras substituted correctly and $|PQ|$ evaluated correctly or consistently

Q9	Model Solution – 55 Marks	Marking Notes
(d) (i)	$$|TS| = 2|QS|$$ $$= 2 \times 80$$ $$= 160$$ $$a^2 = b^2 + c^2 - 2bc\cos\theta$$ $$160^2 = 100^2 + 100^2$$ $$- 2(100)(100)\cos\theta$$ $$20000\cos\theta = 10000 + 10000 - 25600$$ $$\cos\theta = -\frac{5600}{20000}$$ $$\cos\theta = -\frac{7}{25}$$ $$\theta = \cos^{-1} -\left(\frac{7}{25}\right)$$ $$\theta = 106{\cdot}26$$ $$\theta = 106°$$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* - Work of merit - Cosine formula with some substitution *Mid Partial Credit:* - Two variables correctly substituted into the Cosine formula *High Partial Credit:* - Formula substituted correctly - One incorrect substitution followed by correct calculation - Incorrect calculator mode (apply once in paper) *No Credit:* - Treats triangle as right angled
(d) (ii)	$$\text{Arc } TS = 2\pi r \times \frac{\theta}{360}$$ $$= 2\pi(100) \times \frac{106}{360}$$ $$= \frac{530\pi}{9}$$ $$= 185{\cdot}0049$$ $$= 185$$ $$\text{Difference} = 185 - 160$$ $$= 25 \text{ km}$$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* - Work of merit - Arc/circumference formula with some substitution - Reference to $\frac{106}{360}$ *High Partial Credit:* - Calculates arc length $= 185$

Q9	Model Solution – 55 Marks	Marking Notes
(d) **(iii)**	Area of sector RST $= \pi r^2 \times \dfrac{\theta}{360}$ $= \pi(100)^2 \times \dfrac{106}{360}$ $= \dfrac{26500\pi}{9}$ $= 9250\cdot245$ Number of ships $= \dfrac{9250\cdot245}{25}$ $= 370$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit • Formula with some substitution *Mid Partial Credit:* • Formula substituted correctly *High Partial Credit:* • Area of sector evaluated correctly • One error in substitution and finishes correctly or consistently

Blank Page

Blank Page