NICKS & TRICKS

LCOL Guide to – Calculus II

1. Here are the basics of differentiation – don't forget about page 25 of your log book!

Basics of differentiation

Function – $f(x)$	Derivative $f'(x)$
x^n	nx^{n-1}
x^3	$3x^{3-1} = 3x^2$
x^5	$5x^4$
x^{10}	$10x^9$
x^1	$1x^0 = 1$

2. Calculus is very useful for practical questions as well.

Remember: differentiating any equation will give you the equation for its rate of change!

and to find the maximum or minimum of any function, differentiate and put = 0!

Worked Example

The amount, in appropriate units, of a certain medicinal drug in the bloodstream t hours after it has been taken can be estimated by the function:

$C(t)=-t^3+4.5t^2+54t$, where $0 \leq t \leq 9$, $t \in \mathbb{R}$.

(i) Use the drug amount function $C(t)=-t^3+4.5t^2+54t$ to find, in terms of t, the rate at which the drug amount is changing after t hours.

We need to differentiate the function to find the rate of change:

$$C(t) = -t^3 + 4.5t^2 + 54t$$
$$C'(t) = -3t^2 + 9t + 54$$

(ii) Use your answer to part (i) to find the rate at which the drug amount is changing after 4 hours.

Here we sub in t = 4 to our rate of change function

$$C'(t) = -3t^2 + 9t + 54$$
$$C'(4) = -3(4)^2 + 9(4) + 54$$
$$C'(4) = -48 + 36 + 54$$
$$C'(4) = 42$$

After 4 hours, the drug amount is changing at a rate of 42 units per hour.

(iii) Use your answer to part (i) to find the maximum amount of the drug in the bloodstream over the first 9 hours.

Maximum/Minimum
$C'(t) = 0$
and solve for t

$$C(t) = -t^3 + 4.5t^2 + 54t$$
$$C'(t) = -3t^2 + 9t + 54$$

$$-3t^2 + 9t + 54 = 0$$
$$t^2 - 3t - 18 = 0$$
$$(t-6)(t+3) = 0$$

$t - 6 = 0$ $\qquad$ $t + 3 = 0$

$t = 6$ hours $\qquad$ $t = -3$

Not a valid solution

The maximum amount of the drug in the bloodstream occurs after 6 hours.

Now we sub t = 6 into the function to find the maximum amount of drug in the bloodstream.

$$C(t) = -t^3 + 4.5t^2 + 54t$$
$$C(6) = -(6)^3 + 4.5(6)^2 + 54(6)$$
$$C(6) = -216 + 162 + 324$$
$$C(6) = 270 \text{ units}$$

The maximum amount of the drug in the bloodstream over the first 9 hours is 270 units.

(iv) Use your answer to part (i) to show that the drug amount in the bloodstream is decreasing 7 hours after the drug has been taken. Explain your reasoning.

Sub t = 7 into our rate of change function:

$$C'(7) = -3(7)^2 + 9(7) + 54$$
$$C'(7) = -147 + 63 + 54$$
$$C'(7) = -30$$

Rate of Change at $t = 7$ hours

After 7 hours, the drug amount is changing at a rate of – 30 units per hour.

As this is negative the amount of the drug us decreasing.

2017 Question – 22 mins – Time yourself!

Question 8 **(45 marks)**

A company makes and sells fibre optic cable. It can sell, at most, 200 kilometres of cable in a week. For a certain range of its production the company has found that profit can be modelled using the function:

$$P(x) = 275x - x^2 - 2000, \text{ where } x \leq 200.$$

In the function, x is the number of kilometres of fibre optic cable sold and $P(x)$ is the profit in euro.

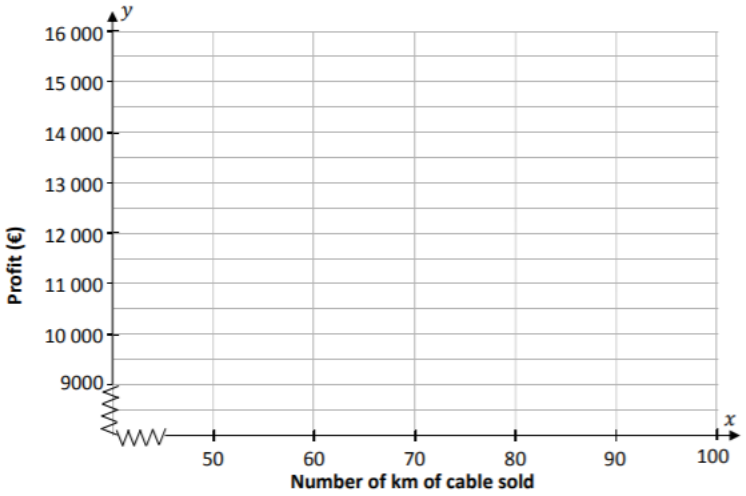
(a) Use the profit function, $P(x)$, to find how much money the company loses if it does not sell any cable.

(b) In a particular week the company made a profit of €8350.
Find the number of kilometres of cable it sold that week.

(c) **(i)** The table below shows some of the data representing the profit on sales.
Use the profit function, $P(x)$, to complete the table.

Number of km of cable sold (x)	50	60	70	80	90	100
Profit (€)			12 350			

(ii) Use the data in the table to draw the graph of the profit function on the axes below for $50 \leq x \leq 100,\ x \in \mathbb{R}$.



(iii) Use your graph to estimate the lower and upper range of sales (in km of cable) in order to make a profit of between €10 000 and €14 000 in a particular week.
Show your work on the graph above.

Lower = Upper =

(d) Use calculus to find the number of kilometres of cable sold when the profit is increasing at a rate of €105 per km.

previous	page	running