

NICKS & TRICKS

LCHL Guide to – Logarithms (Logs)

1. Intro to Logs

$\text{Log}_2 8$	→	Means “The number we put 2 to the power of to get 8” i.e 3
Knowing this, we can work out how to get rid of a Log in question e.g:		
$\text{Log}_2 8 = x$ $2^x = 8$ $2^x = 2^3$ $x = 3$	←	<p>We put the base number in the Log to the power of what's on the other side of the equals, and we bring the big number in the Log over to the other side!</p>
	→	$\text{Log}_x 25 = 2$ $x^2 = 25$ $x = 5$

These are any functions that use “e” or “ln”. These questions are the **same as functions questions**, but they have one thing that **always comes up** that you need to remember!

Whenever you are trying to solve for something that is inside the power of e:

- 1) Get the ln of both sides.
- 2) Now you are allowed to take the power outside the ln.
- 3) $\ln(e) = 1$ so we can get rid of it.
- 4) Solve rest as usual! (You can get what $\ln(20)$ is by putting it into your calculator).

$$e^{10x} = 20$$

$$\ln(e^{10x}) = \ln(20)$$

$$10x \ln(e) = \ln(20)$$

$$10x = \ln(20)$$

$$x = \frac{\ln(20)}{10}$$

$$x = 0.3$$

Worked Example

Question 1

Given $\text{Log}_a 2 = p$ & $\text{Log}_a 3 = q$:

Find $\text{Log}_a \frac{8}{3}$ in terms of p & q

$$\begin{aligned} & \text{Log}_a \frac{8}{3} \\ & \text{Log}_a 8 + \text{Log}_a 3 \quad \text{Page 21 middle column, 1st equation} \\ & \text{Log}_a 2^3 + \text{Log}_a 3 \\ & 3\text{Log}_a 2 + \text{Log}_a 3 \quad \text{Page 21 middle column, 3rd equation} \\ & 3p + q \end{aligned}$$

8 is the same thing as 2^3

Séana agus logartaim	Indices and logarithms
$a^p a^q = a^{p+q}$	$\log_a(xy) = \log_a x + \log_a y$ ★
$\frac{a^p}{a^q} = a^{p-q}$	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ ★
$(a^p)^q = a^{pq}$	$\log_a(x^q) = q \log_a x$ ★
$a^0 = 1$	$\log_a 1 = 0$
$a^{-p} = \frac{1}{a^p}$	$\log_a\left(\frac{1}{x}\right) = -\log_a x$
$a^{\frac{1}{q}} = \sqrt[q]{a}$	$a^{\log_a x} = x$
	$\log_b x = \frac{\log_a x}{\log_a b}$

For any question with logs, just look at **page 21** of the Log Tables (especially the 3 equations I have put stars next to) and try to **manipulate the question in whatever way you can!**

2 Questions – Time yourself!

Question 1

Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q :

$$\log_a \frac{9a^2}{16}$$

Question 2

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon-14, relative to the total carbon content in the item. The formula used is $Q = e^{-\frac{0.693t}{5730}}$, where Q is the proportion of carbon-14 remaining and t is the age, in years, of the item.

- (a) An item is 2000 years old. Use the formula to find the proportion of carbon-14 in the item.
- (b) The proportion of carbon-14 in an item found at Lough Boora, County Offaly, was 0.3402. Estimate, correct to two significant figures, the age of the item.