NICKS & TRICKS

LCHL Guide to – Complex Numbers

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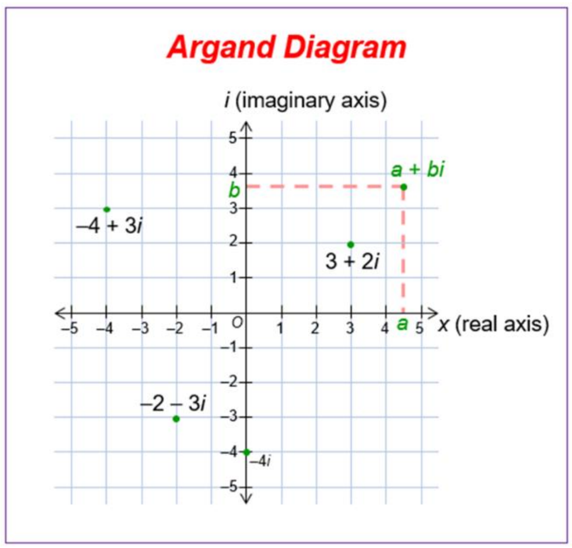
1. To plot complex numbers, treat the real part like an x coordinate and treat the imaginary part like a y coordinate!



2. To convert complex numbers from 'Rectangular' to 'Polar' form:

Given the number $z = a + bi$

1. Identify a and b

2. Find the radius using $r = \sqrt{a^2 + b^2}$

3. Find the angle θ using the formula $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

   [add π whenever $a < 0$]

4. Write in the form $z = r(\cos\theta + i\sin\theta)$

Worked Example

(i) Let $z_1 = 1 - 2i$, where $i^2 = -1$. The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15$.

Find the other two roots of the equation.

When a cubic equation has real co-efficients, two of it's roots will be a conjugate pair:

$$\text{If } 1 - 2i \text{ is a root then so is } 1 + 2i$$

Now we can form a quadratic equation in order to find the 3rd root:

Forming Quadratic Equation Given Roots
$z^2 - (\text{sum of roots})z + (\text{product of roots}) = 0$

$$z^2 - (\text{sum of the roots})z + \text{product of the roots}$$
$$z^2 - (1 - 2i + 1 + 2i)z + (1 - 2i)(1 + 2i)$$
$$z^2 - 2z + 1 + 2i - 2i - 4i^2$$
$$z^2 - 2z + 1 - 4(-1)$$
$$z^2 - 2z + 1 + 4$$
$$z^2 - 2z + 5$$

And using algebraic long division:

$$\begin{array}{r|l} & \quad 2z - 3 \\ \hline z^2 - 2z + 5 & 2z^3 - 7z^2 + 16z - 15 \\ & 2z^3 - 4z^2 + 10z \\ \hline & \qquad -3z^2 + 6z - 15 \\ & \qquad -3z^2 + 6z - 15 \\ \hline & \qquad\qquad\qquad\quad 0 \end{array}$$

$$2z - 3 = 0$$
$$2z = 3$$
$$z = \frac{3}{2}$$

We find our roots to be:

$$\frac{3}{2}, 1 - 2i, 1 + 2i$$

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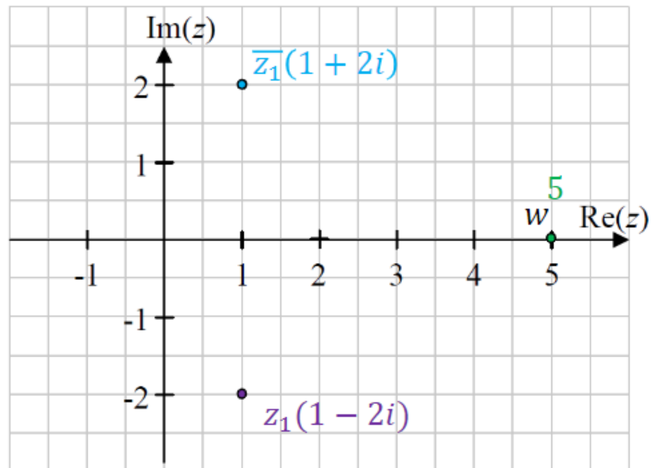
(ii) Let $w = z_1\overline{z_1}$, where $\overline{z_1}$ is the conjugate of z_1. Plot z_1, $\overline{z_1}$ and w on the Argand diagram and label each point.

Tof find the complex conjugate we change the sign on the complex component (the i part)

$$z_1 = 1 - 2i$$

$$\overline{z_1} = 1 + 2i$$

$$w = z_1\overline{z_1}$$
$$w = (1 - 2i)(1 + 2i)$$
$$w = 1(1 + 2i) - 2i(1 + 2i)$$
$$w = 1 + 2i - 2i - 4i^2$$
$$w = 1 - 4(-1)$$
$$w = 1 + 4$$
$$w = 5$$

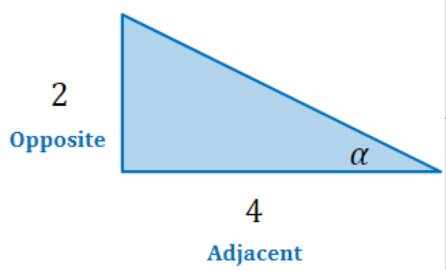


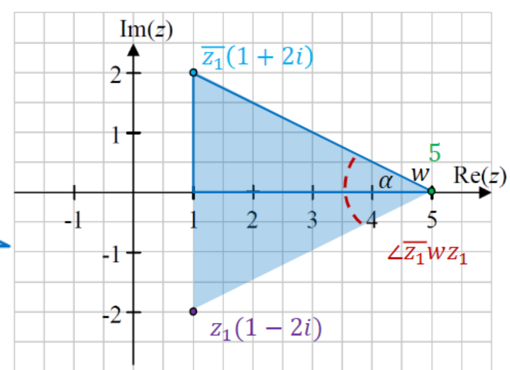
(iv) Find the measure of the acute angle, $< \overline{z_1}wz_1$, formed by joining $\overline{z_1}$ to w to z_1 on the diagram. Give your answer correct to the nearest degree

Using trigonometry, we can find the angle α:

$$\text{Tan} = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan \alpha = \frac{2}{4}$$
$$\alpha = \tan^{-1}\frac{2}{4}$$
$$\alpha = 26.57°$$





Now doubling α gives us the angle $< \overline{z_1}wz_1$:

$$\angle\overline{z_1}wz_1 = 2\alpha$$
$$\angle\overline{z_1}wz_1 = 2(26.57)$$
$$\angle\overline{z_1}wz_1 = 53.14°$$

$$\angle\overline{z_1}wz_1 \approx 53°$$

2019 Question – 12 mins – Time yourself!

Question 5 **(25 marks)**

(a) $3 + 2i$ is a root of $z^2 + pz + q = 0$, where $p, q \in \mathbb{R}$, and $i^2 = -1$.
Find the value of p and the value of q.

(b) **(i)** $v = 2 - 2\sqrt{3}i$. Write v in the form $r(\cos\theta + i\sin\theta)$, where $r \in \mathbb{R}$ and $0 \leq \theta \leq 2\pi$.

(ii) Use your answer to **part (b)(i)** to find the **two** possible values of w, where $w^2 = v$. Give your answers in the form $a + ib$, where $a, b \in \mathbb{R}$.